Lecture 5

Transmission Line II and Matching

October 29, 2002

- Transmission Lines are "Guiding" devices for carrying electromagnetic waves to and from antenna, etc.

- Transmission Line behavior occurs when the wavelength of the wave is small relative to the length of the cable.

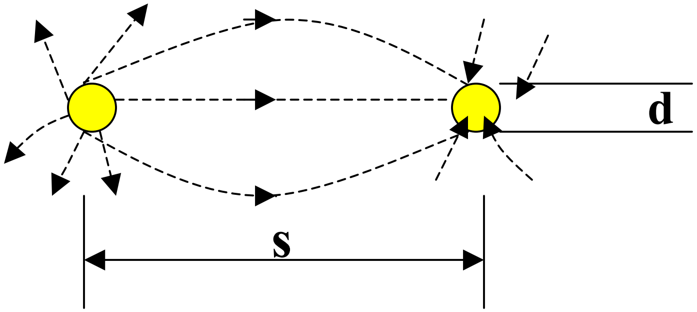
- We have already shown that in a loss-less line (zero resistance along conductors, infinite resistance between them), the propagation speed is given by

$$\nu = \frac{1}{\sqrt{LC}}\left[m/s\right]$$

where L and C are the inductance and capacitance per unit length. The characteristic (specific) impedance is given by:

$$Z_\circ = \sqrt{\frac{L}{C}}\left[\Omega\right]$$

Types of Transmission Lines:

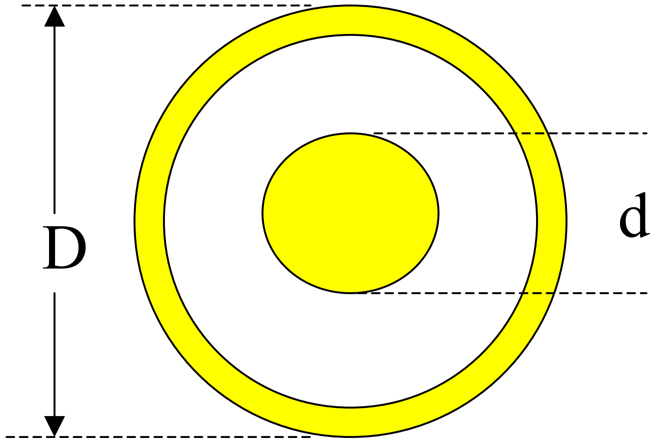


$$Z_\circ = \frac{120}{\sqrt{\varepsilon_r}} ln \frac{2s}{d} [\Omega]$$

$$L = \frac{\mu}{\pi} ln \frac{2s}{d} [H/m]$$

$$C = \frac{\pi\varepsilon}{ln \frac{2s}{d}} [F/m]$$

$$\nu = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} [m/s]$$



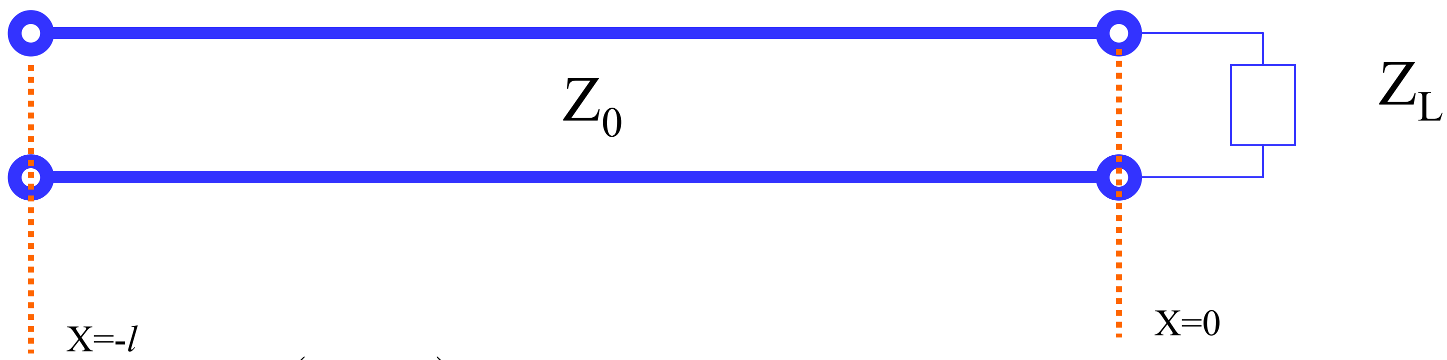
$$Z_\circ = \frac{60}{\sqrt{\varepsilon_r}} ln \frac{D}{d} [\Omega]$$

$$L = \frac{\mu}{2\pi} ln \frac{D}{d} [H/m]$$

$$C = \frac{2\pi\varepsilon}{ln \frac{D}{d}} [F/m]$$

$$\nu = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} [m/s]$$

Transmission Lines with Unmatched Termination:



Incident Wave: $\bar{v}_i e^{j(\omega t-\beta x)}$

Reflected Wave: $\bar{v}_r e^{j(\omega t+\beta x)}$

$$\therefore v(x,t)=\bar{v}_i e^{j(\omega t-\beta x)}+\bar{v}_r e^{j(\omega t+\beta x)}=\tilde{v}(x)e^{j\omega t}\ \textit{ where } \tilde{v}(x)=\bar{v}_i e^{j(-\beta x)}+\bar{v}_r e^{j(\beta x)}$$

Similarly: $i(x,t)=\bar{i}(x)e^{j\omega t}\ \textit{ where } \bar{i}(x)=\frac{\bar{v}_i}{Z_\circ}e^{j(-\beta x)}-\frac{\bar{v}_r}{Z_\circ}e^{j(\beta x)}$

Notice minus sign in reflected current component: Energy flows in opposite direction to incident wave.

Transmission Lines with Unmatched Termination:

$$\text{Now} \quad \frac{\tilde{v}(0)}{\tilde{i}(0)} = Z_L \text{ and } \left.\begin{aligned} \tilde{v}(0) &= \bar{v}_i + \bar{v}_r \\ \tilde{i}(0) &= \frac{\bar{v}_i}{Z_\circ} - \frac{\bar{v}_r}{Z_\circ} \end{aligned}\right\} Z_L = \frac{\bar{v}_i + \bar{v}_r}{\bar{v}_i - \bar{v}_r} \cdot Z_\circ$$

$$\therefore Z_L \bar{v}_i - Z_L \bar{v}_r = Z_\circ \bar{v}_i + Z_\circ \bar{v}_r$$

$$(Z_L - Z_\circ)\bar{v}_i = (Z_L + Z_\circ)\bar{v}_r$$

$$\therefore \frac{\bar{v}_r}{\bar{v}_i} = \text{Reflected Coefficient} \Rightarrow \Gamma = \frac{Z_L - Z_\circ}{Z_L + Z_\circ}$$

Transmission Lines with Unmatched Termination:

$$\tilde{v}(x) = \bar{v}_i \left(e^{-j\beta x} + \Gamma e^{j\beta x} \right)$$

$$\tilde{i}(x) = \frac{\bar{v}_i}{Z_\circ} \left(e^{-j\beta x} - \Gamma e^{j\beta x} \right)$$

Generalized Impedance : $Z(x) = \dfrac{\tilde{v}(x)}{\tilde{i}(x)} = Z_\circ \dfrac{e^{-j\beta x} + \Gamma e^{j\beta x}}{e^{-j\beta x} - \Gamma e^{j\beta x}}$

$$= Z_\circ \frac{e^{-j\beta x} + \dfrac{Z_L - Z_\circ}{Z_L + Z_\circ} e^{j\beta x}}{e^{-j\beta x} - \dfrac{Z_L - Z_\circ}{Z_L + Z_\circ} e^{j\beta x}}$$

Transmission Lines with Unmatched Termination:

$$= Z_\circ \left[\frac{(Z_L + Z_\circ)(\cos\beta x - j\sin\beta x) + (Z_L - Z_\circ)(\cos\beta x + j\sin\beta x)}{(Z_L + Z_\circ)(\cos\beta x - j\sin\beta x) - (Z_L - Z_\circ)(\cos\beta x + j\sin\beta x)} \right]$$

$$= Z_\circ \left[\frac{\begin{array}{l} Z_L\cos\beta x - jZ_L in\beta x + Z_\circ\cos\beta x - jZ_\circ\sin\beta x \\ + Z_L\cos\beta x + jZ_L\sin\beta x - Z_\circ\cos\beta x - jZ_\circ\sin\beta x \end{array}}{\begin{array}{l} Z_L\cos\beta x - jZ_L in\beta x + Z_\circ\cos\beta x - jZ_\circ\sin\beta x \\ - Z_L\cos\beta x - jZ_L\sin\beta x + Z_\circ\cos\beta x + jZ_\circ\sin\beta x \end{array}} \right]$$

$$= Z_\circ \left[\frac{2Z_L\cos\beta x - j2Z_\circ\sin\beta x}{2Z_L\cos\beta x - j2Z_L\sin\beta x} \right] \Rightarrow \boxed{Z(x) = Z_\circ \frac{Z_L - jZ_\circ\tan\beta x}{Z_\circ - jZ_L\tan\beta x}}$$

Impedance "visible" at end of line $Z(-\ell)$

$$\boxed{Z(-\ell) = Z_\circ \frac{Z_L + jZ_\circ\tan\beta\ell}{Z_\circ + jZ_L\tan\beta\ell}}$$

Transmission Lines with Unmatched Termination:

Suppose $\ell \to \lambda/4$ [*Quarter* − *Wavelength*]

so $\beta\ell \to \beta\dfrac{\lambda}{4} = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{4} = \dfrac{\pi}{2}$ *And* $\tan\dfrac{\pi}{2} = \infty$

$$\therefore Z\left(-\frac{\lambda}{4}\right) = \lim_{\ell \to \lambda/4}\left[Z(-\ell)\right] = Z_\circ \frac{jZ_\circ \tan\beta\ell}{jZ_L \tan\beta\ell} = \frac{Z_\circ^2}{Z_L}$$

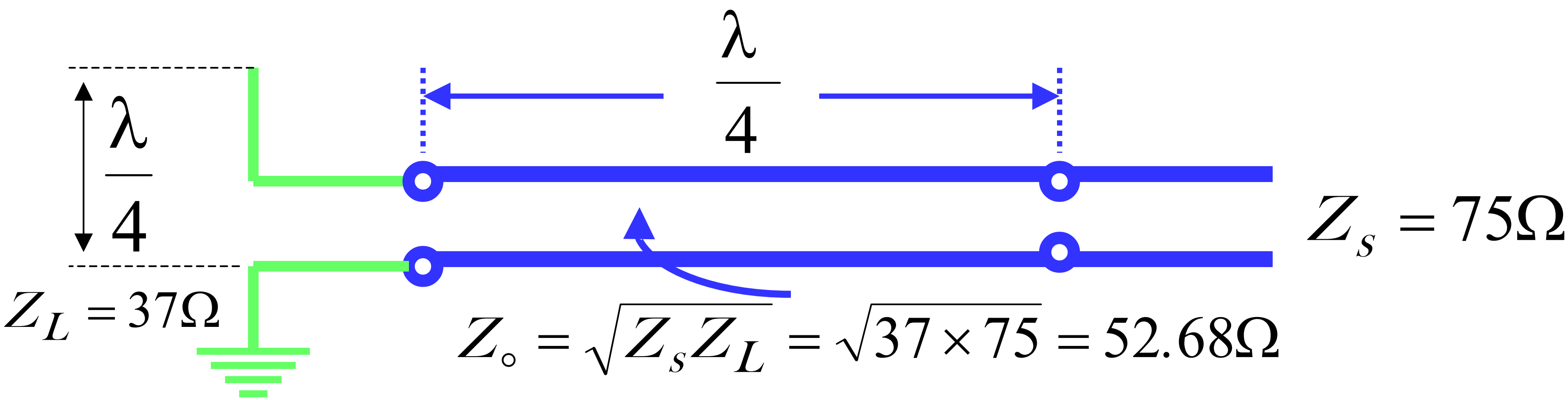
$\therefore$ *We can match a line of impedance* Z_s *to a load of impedance*

Z_L *with a* $\dfrac{\lambda}{4}$ *section of characteristic impedance* $Z_\circ$ *such that*

$$\boxed{Z_\circ = \sqrt{Z_s Z_L}\ \Omega}$$

Transmission Lines :

Example- Suppose we want to match a 75Ω transmission line to a 37 Ω Marconi Antenna.



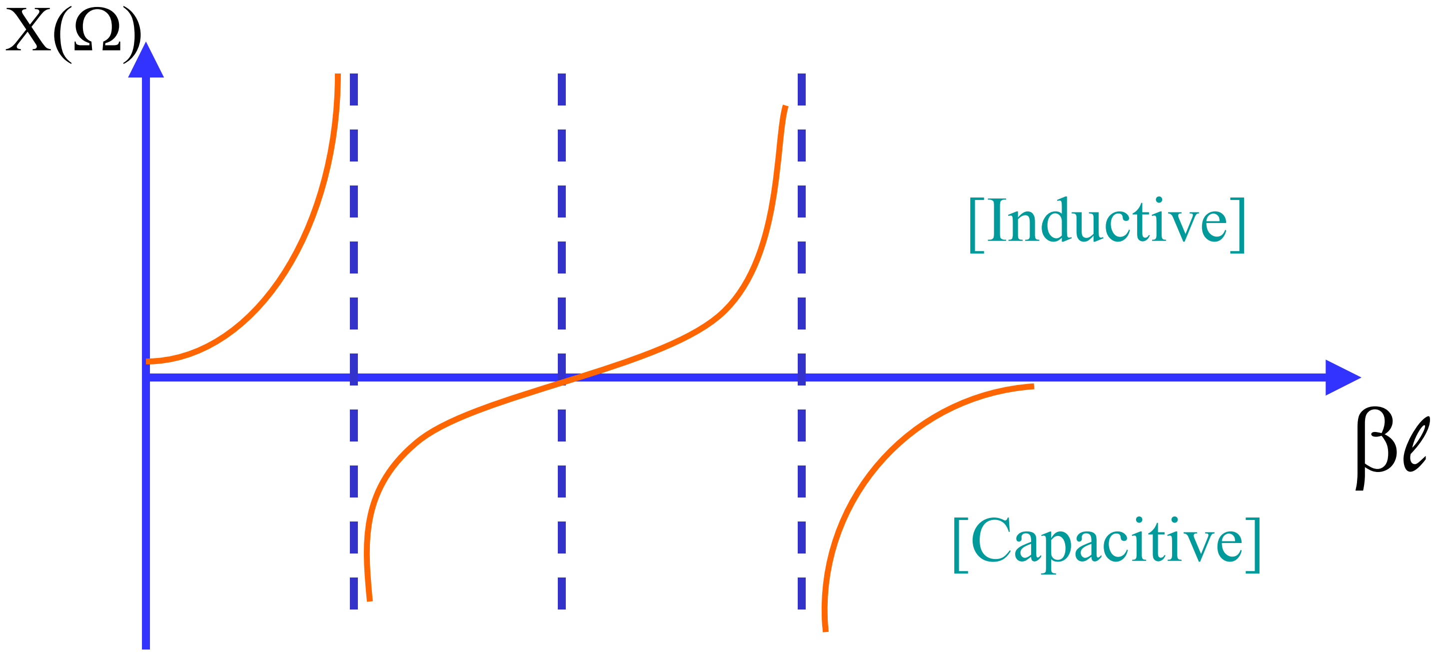
$$Z_\circ = \sqrt{Z_s Z_L} = \sqrt{37 \times 75} = 52.68\Omega$$

The matching cane therefore be achieved using a Quarter-Wavelength section of 50 Ω Transmission Line.

Transmission Lines :

Now consider a short circuit termination: (Z_L=0)

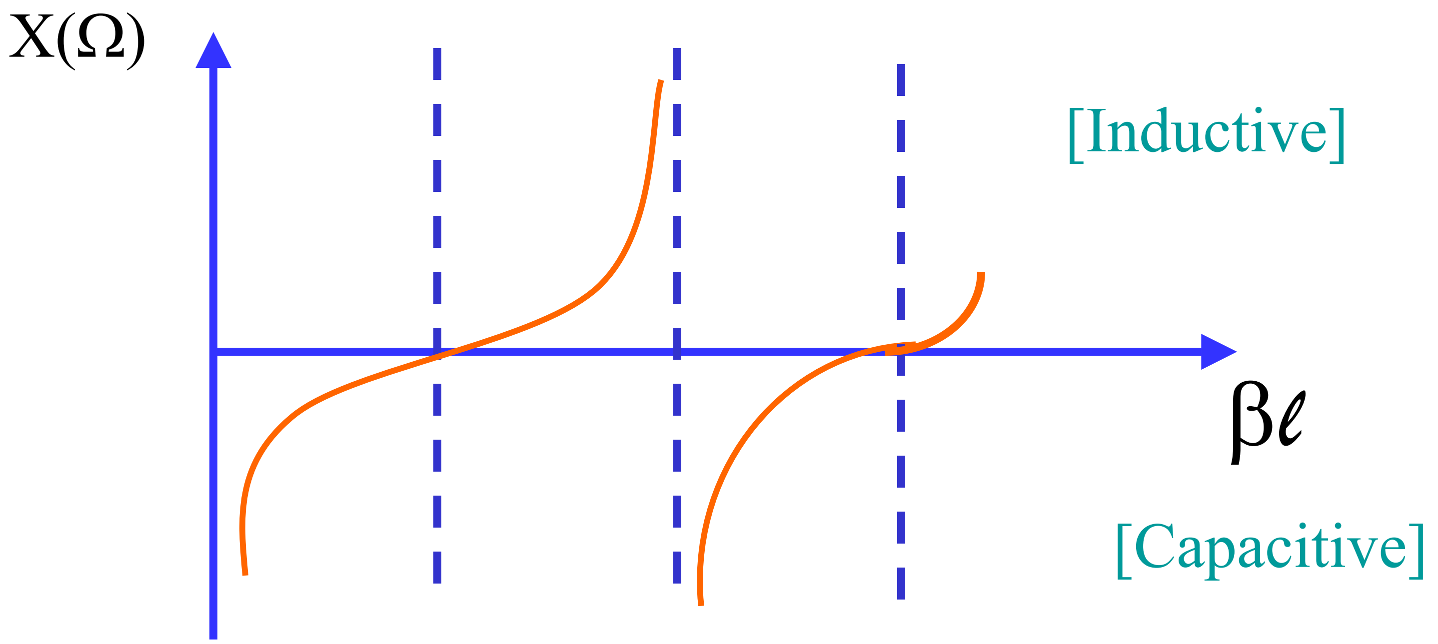
$$\therefore Z(-\ell) = Z_\circ \frac{jZ_\circ \tan\beta\ell}{Z_\circ} = j \underbrace{Z_\circ \tan\beta\ell}_{Reactance\ \mathcal{X}(\Omega)}$$



Transmission Lines :

Now consider an open circuit termination: $(Z_L=\infty)$

$$\therefore Z(-\ell) = Z_\circ \frac{Z_L}{jZ_L \tan\beta\ell} = j(-Z_\circ \cot\beta\ell)$$



standing waves result when a voltage generator of output voltage (V_G = 1sinωt) and source impedance Z_G drive a load impedance Z_L through a transmission line having characteristic impedance Z_0, where $Z_G = Z_0/Z_L$ and where angular frequency ω corresponds to wavelength *l* (b). The values shown in Figure a result from a reflection coefficient of 0.5.

Here, probe A is located at a point at which peak voltage magnitude is greatest—the peak equals the 1-V peak of the generator output, or incident voltage, plus the in-phase peak reflected voltage of 0.5 V, so on your oscilloscope you would see a time-varying sine wave of 1.5-V peak amplitude (trace c). At point C, however, which is located one-quarter of a wavelength (*l/4*) closer to the load, the reflected voltage is 180° out of phase with the incident voltage and subtracts from the incident voltage, so peak magnitude is the 1-V incident voltage minus the 0.5-V reflected voltage, or 0.5 V, and you would see the red trace. At intermediate points, you'll see peak values between 0.5 and 1.5 V; at B (offset *l/8* from the first peak) in c, for example, you'll find a peak magnitude of 1 V. Note that the standing wave repeats every half wavelength (*l/2*) along the transmission line. The ratio of the maximum to minimum values of peak voltage amplitude measured along a standing wave is the standing wave ratio,SWR, SWR=3.

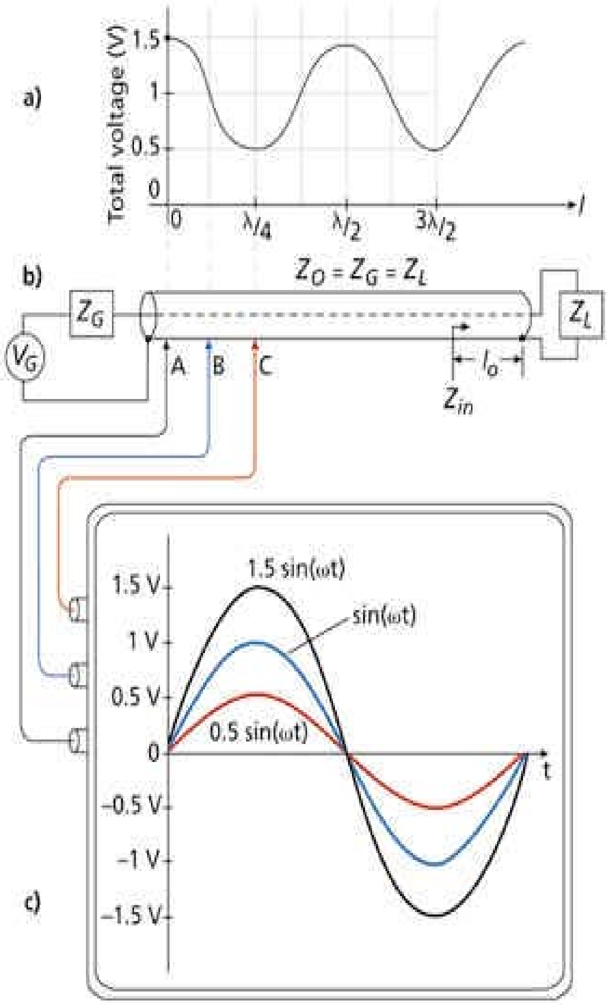
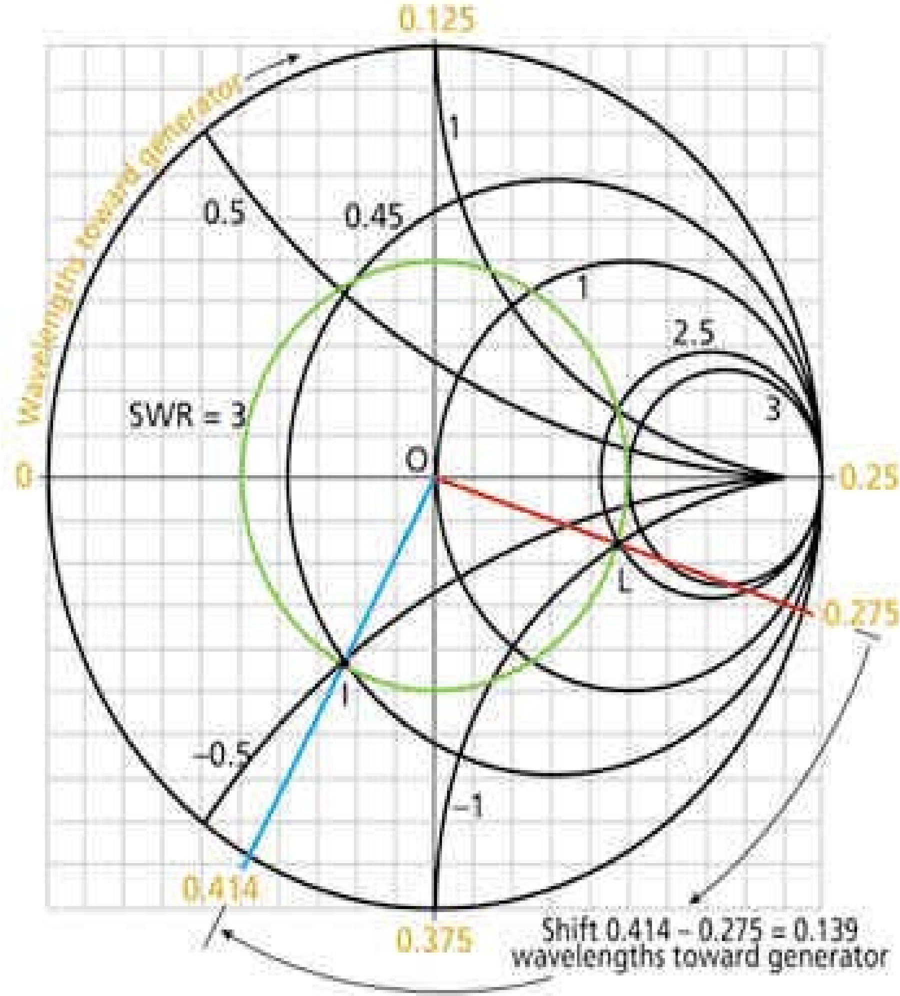


Figure 1

Here, point L represents a normalized load impedance z_L = 2.5 – j1 = 0.5/18° (I chose that particular angle primarily to avoid the need for you to interpolate between resistance and reactance circles to verify the results). The relationship of reflection coefficient and SWR depends only on the reflection coefficient magnitude and not on its phase. If point L corresponds to | G | = 0.5 and SWR = 3, then any point in the complex reflection-coefficient plane equidistant from the origin must also correspond to | G | = 0.5 and SWR = 3, and a circle centered at the origin and whose radius is the length of line segment OL represents a locus of constant-SWR points. (Note that the SWR = 3 circle here shares a tangent line with the r_L = 3 circle at the real axis; this relationship between SWR and r_L circles holds for all values of SWR.)



Using the standing-wave circle, you can determine input impedances looking into any portion of a transmission line such as Figure 1's if you know the load impedance. Figure 1, for instance, shows an input impedance Z_{in} to be measured at a distance l_0 from the load (toward the generator). Assume that the load impedance is as given by point L in Figure 2. Then, assume that l_0 is 0.139 wavelengths. (Again, I chose this value to avoid interpolation.) One trip around the Smith chart is equivalent to traversing one-half wavelength along a standing wave, and Smith charts often include 0- to 0.5-wavelength scales around their circumferences (usually lying outside the reflection-coefficient angle scale previously discussed).

Such a scale is show in yellow in Figure 2, where clockwise movement corresponds to movement away from the load and toward the generator (some charts also include a counter-clockwise scale for movement toward the load).

Using that scale, you can rotate the red vector intersecting point L clockwise for 0.139 wavelengths, ending up at the blue vector. That vector intersects the SWR = 3 circle at point I, at which you can read Figure 1's input impedance Z_{in}. Point I lies at the intersection of the 0.45 resistance circle and –0.5 reactance circle, so Z_{in} = 0.45 – j0.5.

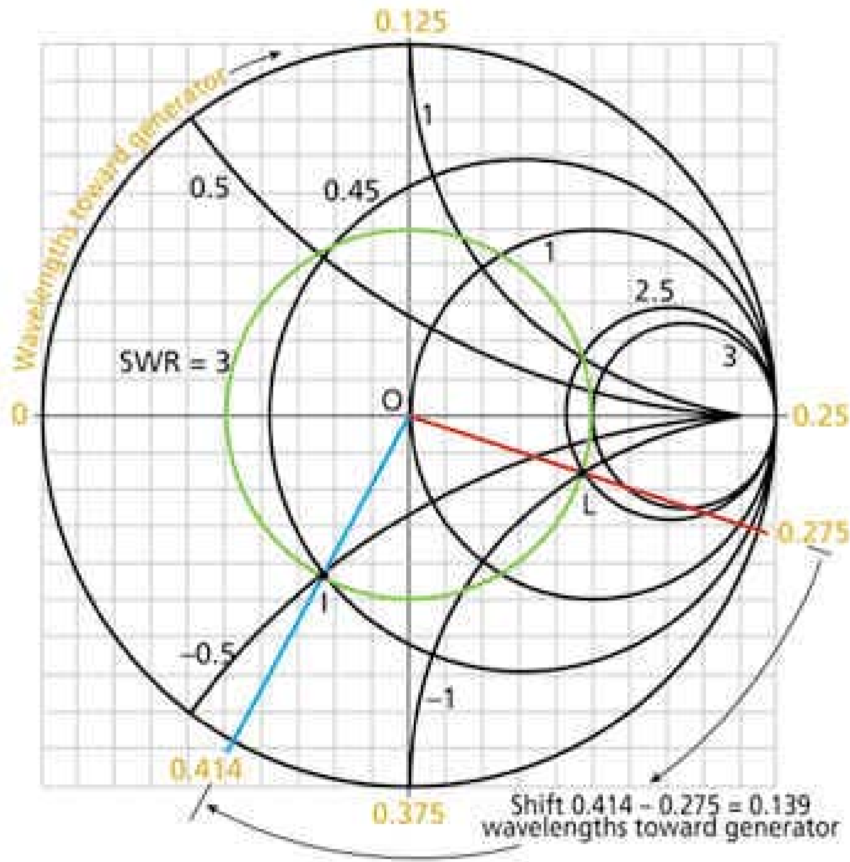


Figure 2

Smith Chart 1

Suppose we have a load with an impedance of (40+j80)Ω which we need to match to a 100 Ω transmission line. (c 3x10[8]m/s and f=130 MHz).

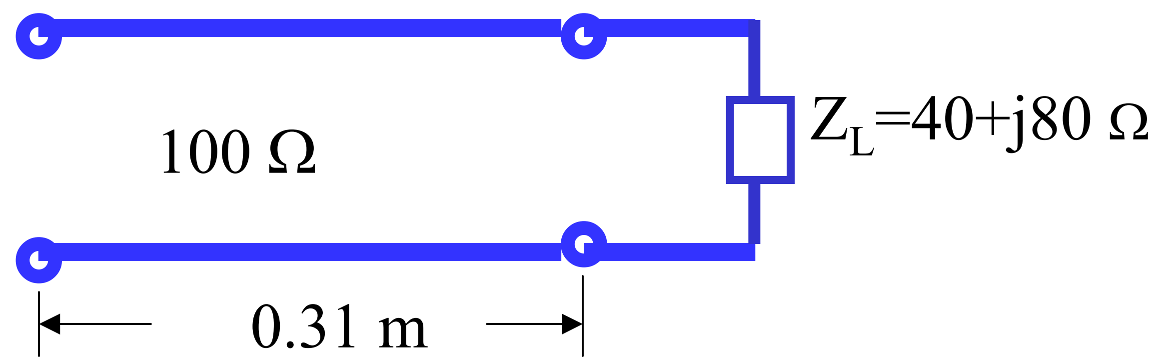
$$Z_{L(n)} = \frac{Z_L}{Z_\circ} = \frac{40 + j80\Omega}{100\Omega} = 0.4 + j0.8$$

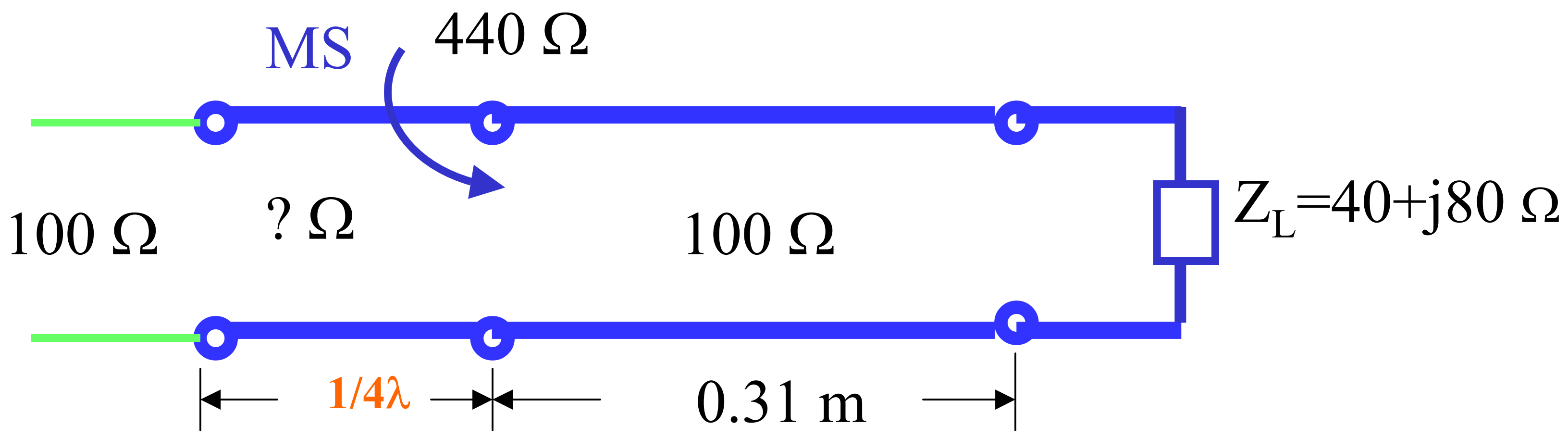
From the Smith Chart, the distance (in wavelength)

from this point toward the generator is $\sim 0.135\lambda$.

The distance is $d = 0.135 \times \dfrac{3\times 10^{8}\, m/s}{130\times 10^{6}\, Hz} = 0.31m$

Looking in here, we see a resistive load of 440 Ω . We can match this to 100 Ω using a 1/4λ matching section.





$$Z_{\circ} = \sqrt{Z_s Z_L}$$

$$Z_{\circ} = \sqrt{100 \times 440} = 209.8\Omega$$

Smith Chart 2

Find the length, position and characteristic impedance of the quarter-wavelength transformer required to match an antenna with an impedance of (30-j40)Ω to a 75 Ω transmission line. The operating frequency of this system is 100MHz and the insulator in the transmission line has a dielectric constant of 10.

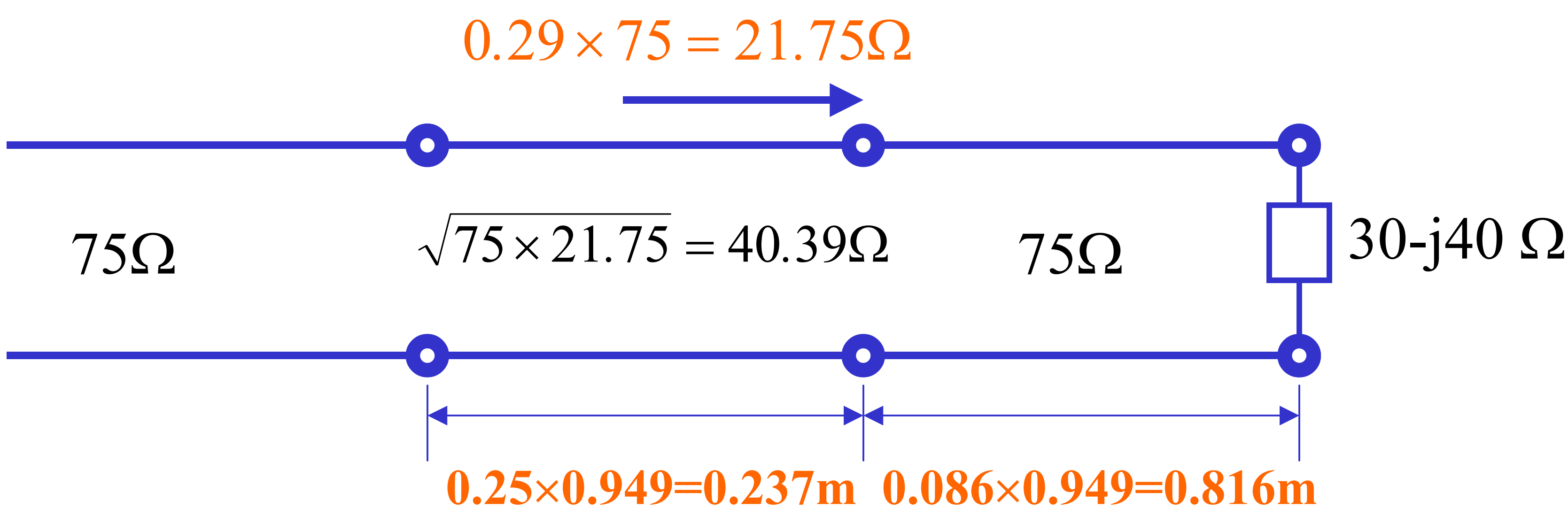
The speed of Wave Propagation $\nu = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8\, m/s}{\sqrt{10}} = 9.49\times 10^7\, m/s$

Wavelength $\lambda = \frac{\nu}{f} = \frac{9.49\times 10^7}{100\times 10^6} = 9.49\, m$

Normalized Impedance $Z_{L(n)} = \frac{Z_L}{Z_\circ} = \frac{30 - j40}{75} = 0.4 - j0.533$

Smith Chart 2

From the Smith Chart, we find that we need to move 0.86λ away from the load (i.e., towards the generator) in order to eliminate the reactance (or imaginary) impedance component. At this point, the resistive (real) element is 0.29. Thus:



Smith Chart 3

Given that $Z_L=(30+j40)\Omega$, $Z_0=50\ \Omega$, find the shortest l and Z_T so that the above circuit is matched. Assume that Z_T is real and loss-less.

We want Z_l to be real and Z_{in} to be $Z_0=50\ \Omega$ in order for Z_T to be real and the matching condition is satisfied. We find that $Z_{L(n)}$ is 0.6+j0.8. In order to make $Z_{l(n)}$ real, the shortest l from the Smith Chart is $\lambda/8$. Then $Z_{l(n)}=3.0$, and $Z_L=150\ \Omega$. Since $Z_{in}=50\ \Omega$, we need

$$Z_T=\sqrt{Z_{in}Z_\ell}=\sqrt{50\times 150}=86.6\Omega$$