LS-125
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September 1988

MODE SELECTION AND BOUNDARY CONDITIONS USING MAFIA IN FREQUENCY-DOMAIN

The field solvers of the 3D code MAFIA in the frequency domain are R3 and E3: the former generates an eigenvalue equation, and the latter solves this equation for eigenfrequencies and eigenvectors (fields). They are usually expensive to use. In order to save memory space and CPU time, one may employ a part of a structure in calculations if there are certain symmetries that are embedded in the geometry. In this case, one must run R3 and E3 several times with different boundary conditions if one wants to get a complete set of modes of the whole structure. However, sometimes only some modes are of interest to us (e.g., TM_{01}), and we may then specify the appropriate boundary conditions for a part of a structure to obtain these modes. There are two types of boundary conditions that one may choose from in R3: either a zero tangential E-field, denoted by integer 1, or a zero tangential H-field, denoted by integer 2. This note will use a circularly cylindrical cavity as an example to discuss the relation between mode selection and boundary conditions when using a part of the cavity.

The cavity is shown in Fig. 1a. The radius is 2 cm and the length 20 cm. The cavity has two 0.4-cm radius beam ports with a length of 2 cm each. We first use an octant of the cavity (Fig. 1b) and run a total of eight times with the following boundary conditions for R3:

(1)		(2)		(3)		(4)	
1	1	2	1	1	1	1	1
1	1	1	1	2	1	1	1
1	1	1	1	1	1	2	1

(5)		(6)		(7)		(8)	
2	1	2	1	1	1	2	1
2	1	1	1	2	1	2	1
1	1	2	1	2	1	2	1

Every set of boundary conditions has three pairs of digits that indicate the boundary conditions to be applied at the lowest and highest values of the x, y, and z coordinates.

The modes obtained in each run are, respectively:

(1) TE_{212}, TE_{214}, TE_{216}, TE_{218}......
(2) TE_{112}, TE_{114}, TE_{116}, TE_{118}......
(3) TE_{112}, TE_{114}, TE_{116}, TE_{118}......
(4) TE_{211}, TE_{213}, TE_{215}, TE_{217}......
(5) TM_{010}, TM_{012}, TM_{014}, TM_{016}......
(6) TE_{111}, TE_{113}, TE_{115}, TE_{117}......
(7) TE_{111}, TE_{113}, TE_{115}, TE_{117}......
(8) TM_{011}, TM_{013}, TM_{015}, TM_{017}......

The frequencies and first few modes are listed in Table 1. The first column of Table 1 lists the complete set of the lowest modes that are obtained when a whole structure is used in calculation with the boundary conditions (1).

These results are easy to understand. For example, in case (1), because the x=0 plane and y=0 plane are perfect conductors, the E-fields are perpendicular to both planes. In addition, the E-fields are also perpendicular to the cylinder wall. Therefore, in the x-y plane, the E-field is a quadrupole (or higher multipoles), and there is no E-field in the longitudinal direction. These are TE_{21} modes (see Fig. 2a). In case (5), the tangential H-field = 0 in the x=0 plane and y=0 plane, and the tangential E-field = 0 on the cylinder wall. So circular H-fields in the x-y plane are possible, and there are no H-fields in the z direction (see Fig. 2b). In this case, the TM_{01} modes will be generated. In cases (2), (6) and (3), (7), the E-fields are shown in Fig. 2C and Fig. 2D, respectively. They are TE_{11} modes. Hybridization has been observed for higher order modes due to the existence of the beam ports. This has not yet been studied in detail.

Next, we use a quarter of the cavity. There are two possible geometries, as shown in Fig. 3 and Fig. 4, respectively. For each geometry, we ran MAFIA four times. The boundary conditions and the corresponding modes are listed below.

For the structure in Fig. 3, the boundary conditions are:

(I)		(II)		(III)		(IV)	
1	1	2	1	1	1	2	1
1	1	1	1	2	1	2	1
1	1	1	1	1	1	1	1

and the lowest modes are:

(I) TE_{211}, TE_{212}, TE_{213}, TE_{214},......
(II) TE_{111}, TE_{112}, TE_{113}, TE_{114},......
(III) TE_{111}, TE_{112}, TE_{113}, TE_{114},......
(IV) TM_{010}, TM_{011}, TM_{012}, TM_{013},......

Their frequencies are listed in Table 2.

In the case of Fig. 4, the boundary conditions are:

(A)		(B)		(C)		(D)	
1	1	2	1	1	1	2	1
1	1	1	1	1	1	1	1
1	1	1	1	2	1	2	1

and the lowest modes are:

(A) TE_{112}, TE_{114}, TE_{116}, TE_{118}, TE_{212},......
(B) TE_{112}, TE_{114}, TM_{010}, TM_{012}, TE_{116},......
(C) TE_{111}, TE_{113}, TE_{115}, TE_{117}, TE_{211},......
(D) TE_{111}, TE_{113}, TE_{115}, TM_{011}, TM_{013},......

Their frequencies are listed in Table 3.

The frequencies listed in the three tables are calculated by E3 and are about 1-2% lower than those obtained by using analytical formula for the same circularly cylindrical cavity without beam ports. This is probably due to the rather rough mesh that was used in our calculations (a total number of mesh points of about 10,000 for the whole cavity). The frequency shift due to the existence of the beam ports is negligible.

We noticed that there are two solutions for the same mode when one obtains the complete modes. The frequencies of the two solutions are either equal to each other, or very near. This means that the program cannot recognize the circular boundary conditions.

Table 1
The lowest modes under each boundary condition for an octant of a cylindrical cavity.

Mode	Frequency MHz	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
TE-111	4402.71						X	X	
TE-112	4588.09		X	X					
TE-113	4876.76						X	X	
TE-114	5245.33		X	X					
TE-115	5670.15						X	X	
TM-010	5701.70					X			
TM-011	5751.52								X
TM-012	5894.71					X			
TM-013	6122.19								X
TE-116	6130.30		X	X					
TM-014	6419.77					X			
TE-117	6608.31						X	X	
TM-015	6771.60								X
TE-118	7090.03		X	X					
TE-211	7150.63				X				
TE-212	7270.81	X							
TE-213	7443.57				X				
TE-214	7693.21	X							

Table 2

The lowest modes under each boundary condition for a quarter of a cylindrical cavity.*

Mode	Frequency MHz	(I)	(II)	(III)	(IV)
TE-111	4402.71		X	X	
TE-112	4588.09		X	X	
TE-113	4876.76		X	X	
TE-114	5245.33		X	X	
TE-115	5670.15		X	X	
TM-010	5701.70				X
TM-011	5751.52				X
TM-012	5894.71				X
TM-013	6122.19				X
TE-116	6130.30		X	X	
TM-014	6419.77				X
TE-117	6608.31		X	X	
TM-015	6771.60				X
TE-118	7090.03		X	X	
TE-211	7150.63	X			
TE-212	7270.81	X			
TE-213	7443.57	X			
TE-214	7693.21	X			

*See Fig. 3.

Table 3

The lowest modes under each boundary condition for a quarter of a cylindrical cavity.*

Mode	Frequency MHz	(A)	(B)	(C)	(D)
TE-111	4402.71			X	X
TE-112	4588.09	X	X		
TE-113	4876.76			X	X
TE-114	5245.33	X	X		
TE-115	5670.15			X	X
TM-010	5701.70		X		
TM-011	5751.52				X
TM-012	5894.71		X		
TM-013	6122.19				X
TE-116	6130.30	X	X		
TM-014	6419.77		X		
TE-117	6608.31			X	X
TM-015	6771.60				X
TE-118	7090.03	X			
TE-211	7150.63			X	
TE-212	7270.81	X			
TE-213	7443.57			X	
TE-214	7693.21	X			

*See Fig. 4.

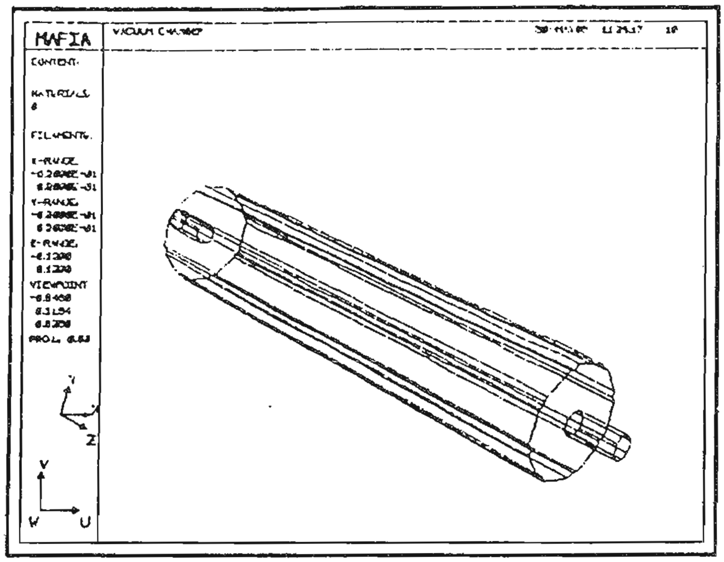

Fig. 1a. A circularly cylindrical cavity with two beam ports.

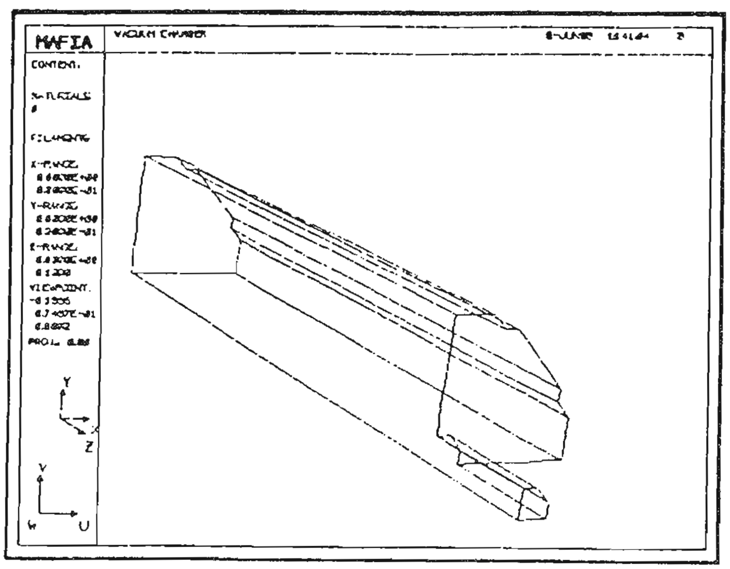

Fig. 1b. An octant of the cavity in Fig. 1a.

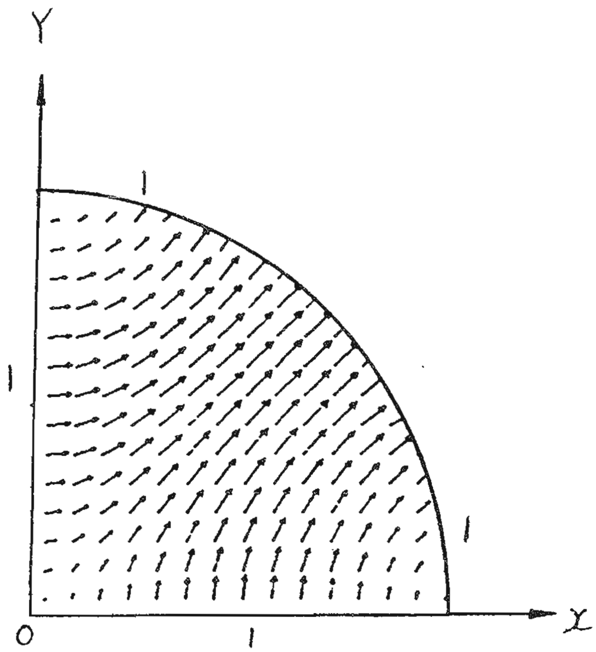

Fig. 2a. $\vec{E}$ field (1), (4)
$\vec{E}$ is perpendicular to x=0 plane, y=0 plane, and cylinder wall for boundary conditions (x,y)

1	1
1	1

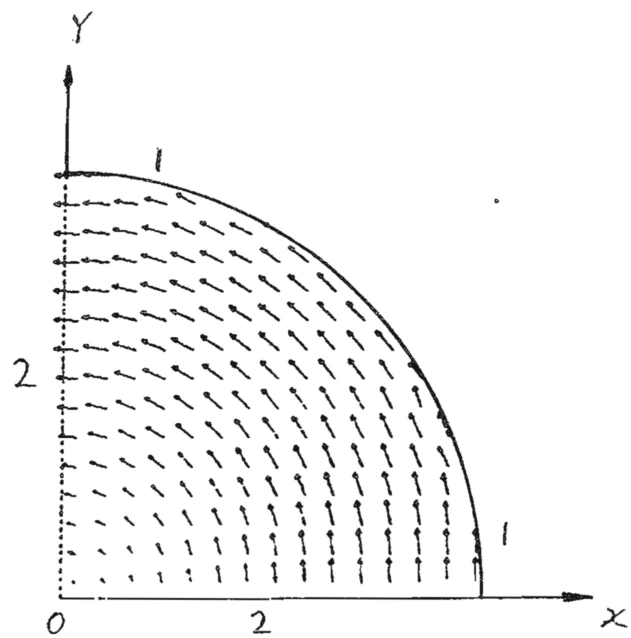

Fig. 2b. $\vec{H}$ field (5), (8)
$\vec{H}$ is perpendicular to x=0 plane, y=0 plane for boundary conditions (x,y)

2	1
2	1

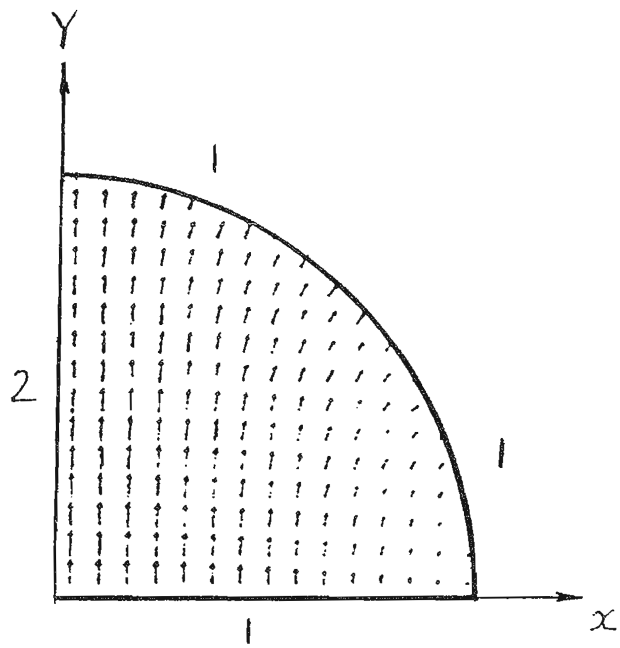

Fig. 2c. $\vec{E}$ field (2), (6)
$\vec{E}$ is perpendicular to y=0 plane for boundary conditions (x,y)

2	1
1	1

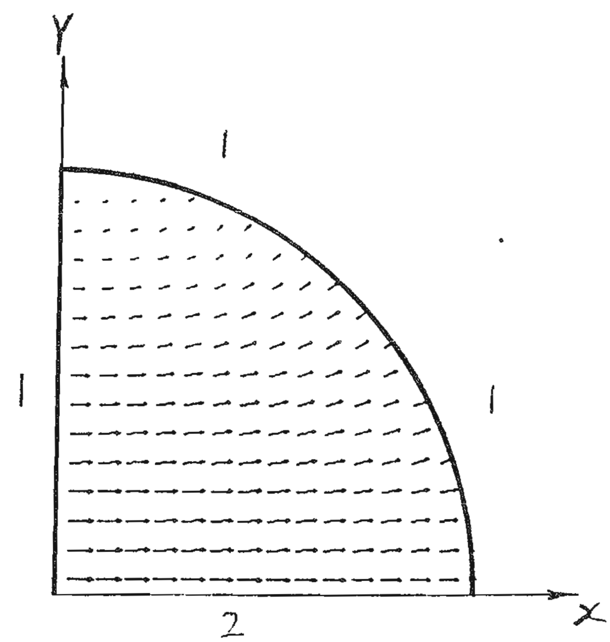

Fig. 2d. $\vec{E}$ field (3), (7)
$\vec{E}$ is perpendicular to x=0 plane for boundary conditions (x,y)

1	1
2	1

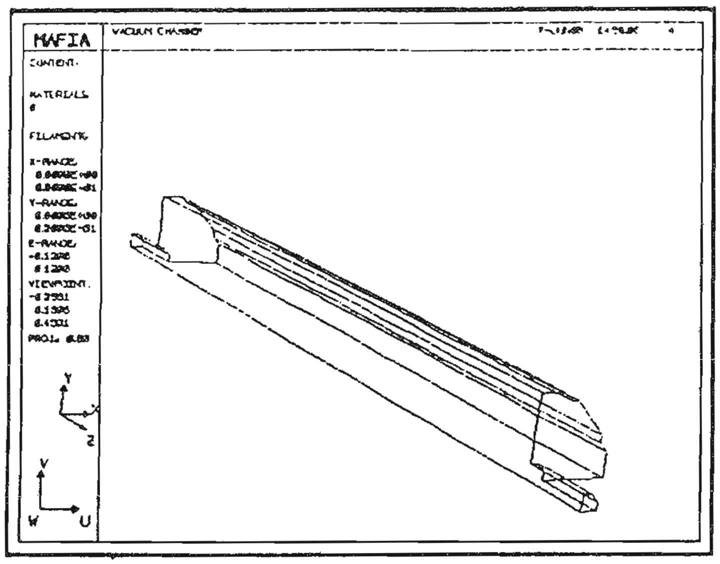

Fig. 3. A quarter of the cavity in Fig. 1a.

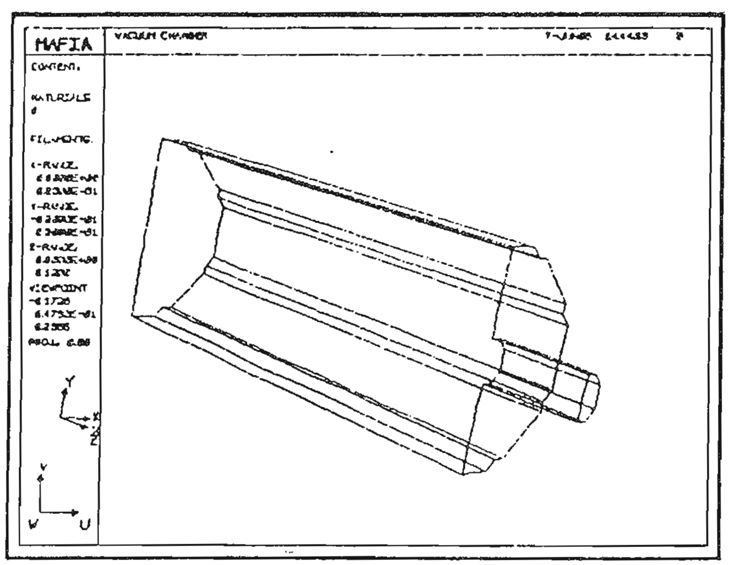

Fig. 4. A quarter of the cavity in Fig. 1a.